

The effect of primordial fluctuations on neutrino oscillations

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Abstract

Recent work has shown that neutrino oscillations in matter can be greatly enhanced by flips between mass eigenstates if the medium is fluctuating with a period equal to the neutrino oscillation length. Here we investigate the effect of the primordial fluctuations on the neutrino oscillations in the early universe. We calculate the oscillation probability in the case of a general power law fluctuation spectrum and for a more realistic spectrum predicted by inflation. We also include the effect of the amplification of fluctuations resulting from the QCD phase transition. We find that there is a region of parameter space where this mechanism would be the dominant mechanism for producing sterile neutrinos. However this conclusion does not take account of the damping of fluctuations on the neutrino oscillation scale when the neutrinos decouple from the plasma. We find that this reduces the probability of flips between the mass eigenstates to an unobservable level.

1 Introduction

For many years there has been tremendous success in determining the masses and mixing of the neutrino sector, including experiments on solar [1], reactor [2, 3], atmospheric [4] and accelerator [5, 6] neutrinos. At the 3σ level the masses and mixing angles determining the solar neutrino oscillations are $\Delta m_{21}^2 = (7.2 - 9.2) \times 10^{-5} \text{ eV}^2$ and $\sin^2 \theta_{12} = 0.25 - 0.39$ [7], while those determining the atmospheric oscillations are $\Delta m_{31}^2 = 2.0 - 3.2 \times 10^{-3}$ and $0.34 < \sin^2 \theta_{23} < 0.68$ with $\sin^2 2\theta_{23} < 0.9$ [7]. Currently the sign of Δm_{23}^2 is undetermined and the current 3σ upper bound on the allowed values for the final mixing angle is $\sin^2 \theta_{13} < 0.044$ [7]. The LSND experiment showed oscillations on a mass scale of $\Delta m^2 \sim 1 \text{ eV}^2$ [8] if this observation is interpreted as standard neutrino oscillations then it can only be explained as oscillations into a fourth neutrino state. Precision measurements at LEP have shown that there are only three neutrinos that interact via the standard weak interaction exist with mass less than $M_Z/2$ and therefore these additional states must be sterile [9]. The global data including the LSND (but not including MiniBoone) comprehensively disfavored the inclusion of just one sterile neutrino [10], the inclusion of 2 sterile neutrinos mildly mixed with the 3 active neutrinos has a more acceptable fit to all data including LSND [11] although the value of the associated LSND mixing angle is still problematic [12]. The inclusion of the MiniBoone experiment did not observe an oscillation [13] but does not rule out this scheme [14]. Also keV sterile neutrinos which naturally arise in many models [15] can also explain pulsar kicks [16] and are warm dark matter candidates [17]. The latter has recently been received interest following the observations of the central cores of low mass galaxies [18] and from the low number of satellites observed in Milky-Way size galaxies [19].

If sterile neutrinos do mix with the standard model neutrinos then they would be produced via oscillations in the early universe [20]. If the sterile neutrinos were produced before the active neutrinos decouple ($T \gtrsim \text{MeV}$)

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active neutrinos would oscillate into sterile neutrinos, then the active neutrinos would be repopulated, resulting in an increase in the number of relativistic degrees of freedom. The expansion rate of the universe would increase, which would lead to a higher freeze-out temperature and therefore a higher neutron to proton ratio, leading to a higher abundance of helium and other elements [21]. This increased expansion rate can also be probed by observations of the CMB [22, 23]. These sterile neutrinos would also contribute to the warm dark matter content of the universe which would suppress the formation of large scale structure [23, 24].

Neutrino oscillations in the early universe have been extensively investigated and bounds have been placed on the mixing angles and difference in mass squared [20]. However, these studies have neglected the fact that there are temperature fluctuations in the early universe. It has been shown that matter fluctuations can cause level crossing between mass eigenstates [25], resulting in an amplification of these oscillations. In the case of active-sterile mixing this could lead to an amplification in the production of sterile neutrinos. If the spectrum of the fluctuations was known and the fluctuations were large enough then new bounds could be placed on the masses and mixing of the neutrino sector. Alternatively if the masses and mixing was known then then constraints could be placed on the spectrum of the fluctuations and on the models which predict them.

This paper is organized as follows. We begin by calculating the level crossing probability of a two neutrino system in an expanding universe with a general spectrum of temperature fluctuations. In sections 3 and 4 we calculate the level crossing probability for the case of a power law spectrum and a spectrum predicted by inflation respectively. In section 5 we investigate the effect of neutrino decoupling and in section 6 the effect of the QCD phase transition. Finally we conclude in section 7.

2 Neutrino evolution

To consider the Neutrino oscillations in a fluctuating media we follow the analysis developed in [25]. The evolution of a neutrino system is determined by the Schrödinger equation, $i\frac{\partial}{\partial t}\nu = H^0(t)\nu$ ¹, where t is time, here H^0 and ν are the Hamiltonian and neutrino wavefunction in flavour basis. This can be rotated such that the hamiltonian is instantaneously diagonalised², $i\frac{\partial}{\partial t}\nu_m^0 = H_m^0\nu_m^0$, where $\nu_m^0 = U\nu$ are the instantaneous mass eigenstates. For a two neutrino system

$$U(t) = \begin{pmatrix} \cos\theta_M & \sin\theta_M \\ -\sin\theta_M & \cos\theta_M \end{pmatrix}, \quad (1)$$

$$H_m(t) = \begin{pmatrix} -\Delta_m & -i d\theta_m/dx \\ i d\theta_m/dx & \Delta_m \end{pmatrix}, \quad (2)$$

where $\Delta_m = \sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin^2 2\theta)^2}/4E$, $\tan 2\theta_m = \delta m^2 \sin 2\theta/(\Delta m^2 \cos 2\theta - A)$, θ is the vacuum mixing angle, Δm^2 is the difference in mass squared, E is the energy of the neutrino and A is the difference in matter potentials of the two neutrino system. Before the neutrinos have decoupled, for temperatures, $T \gtrsim MeV$ the matter potential takes the form

$$A \simeq -\Delta C_v G_F^2 E^2 T^4 / \alpha \quad (3)$$

$$\simeq -1.86 \times 10^{-20} \Delta C_v \left(\frac{E}{MeV} \right)^2 \left(\frac{T}{MeV} \right)^4 MeV^2, \quad (4)$$

where α is the fine structure constant and $C_v = 14\pi (a - \sin^2 \theta_W) \sin^2 \theta_W / 45$, $a = 3$ for ν_e and $\bar{\nu}_e$, $a = 1$ for ν_μ , $\bar{\nu}_\mu$, ν_τ and $\bar{\nu}_\tau$ and $C_v = 0$ for sterile neutrinos. We have assumed that the neutrino, anti-neutrino asymmetry is of the same order as the baryon asymmetry and therefore the contribution to the matter potential can be neglected. There is a resonance when the neutrino system is maximally mixed, when $A = \Delta m^2 \cos 2\theta$. As the matter potential is negative for both neutrinos and anti-neutrinos there will only be a resonance if the difference in mass squared is negative. It is convenient to consider the hamiltonian as the sum of the averaged

¹Here we use the notation H^0 for a hamiltonian in a medium which is not fluctuating.

²where $U^\dagger H^0 U$ is diagonal

Hamiltonian, H^0 and the fluctuated Hamiltonian, δH , where the true hamiltonian, $H \equiv H^0 + \delta H$ and in the absence of fluctuations $\delta H = 0$. The Schrodinger equation is rotated to diagonalize the averaged Hamiltonian where the fluctuations are teated as a perturbation. Now $i\frac{\partial}{\partial t}\nu_m^0 = H_m(t)\nu_m^0$, $H_m(t) = H_m^0 + \delta H_m$ where H_m^0 is given by Eq. (2) and

$$\delta H_m(t) = \frac{\delta V}{2} \begin{pmatrix} -\cos 2\theta_m & \sin 2\theta_m \\ \sin 2\theta_m & \cos 2\theta_m \end{pmatrix}, \quad (5)$$

where $V = A/2E$ and $\delta V = V - V_0$, where $V_0 = A_0/2E$ and A_0 is the matter potential in the absense of fluctuations. If the off diagonal elements are non-zero transitions between the mass eigenstates can occur. If $\Delta_m \ll |d\theta_m/dt|$ the neutrino propagation is non-adiabatic with or without fluctuations. If $|d\theta_m/dt| \ll \Delta_m$ the evolution in the absence of fluctuations is adiabatic and the level crossing between the mass eigenstates is determined by the off diagonal elements in Eq (5), which are $\delta V \sin 2\theta/2$. In the perturbative limit where the fluctuations are small the level crossing probability is

$$P \simeq \left| \int_{t_i}^{t_f} dt \frac{\delta V \sin 2\theta_m}{2} \exp \left(i \int^t dt' 2\Delta_m \right) \right|^2. \quad (6)$$

Numerical simulations have shown that for large fluctuations the system becomes depolarized, i.e. with the probability of detecting each flavour of neutrino being 1/2; a rough criteria for depolarization is the region $P \gtrsim 1/2$. The neutrinos that we consider are in thermal equilibrium with the plasma and therefore have an energy spectrum which is approximately Fermi-Dirac and is charectorised by the temperature of the plasma. For this spectrum the average energy of the neutrinos $\langle E \rangle = yT$, where $y \simeq 3.1514$. For the rest of this paper we consider the oscillations of a neutrino with an energy equal to the average energy of the ensemble of neutrinos, i.e. $E = yT$. To calculate the fluctuated hamiltonian for a neutrino with this energy we use Eq.(3) $V = A/2E \propto ET^4$, in the case of small fluctuations $\delta V = 4V\delta T/T + V\delta E/E$. If the neutrino is in thermal equilibrium with the fluctuations $\delta E/E = \delta T/T$ and therefore $\delta V = 5V\delta T/T$. If the neutrino is not in thermal equilibrium with the fluctuations then $\delta E/E \simeq 0$, i.e. the energy of the neutrino remains approximately constant across the scale of the fluctuation. As we will see in the section 5 the neutrinos decouple from the fluctuations at the scale which amplifies the neutrino oscillations before the time at which the neutrinos oscillate. Therefore it is a good approximation to use assume that the neutrinos are not in thermal equilibrium with the fluctuations and $\delta V = 4V\delta$. Expanding the fluctuations into their Fourier components this now becomes

$$P = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \mathcal{P}(k) \left| \int_{t_i}^{t_f} dt 2V \sin 2\theta_m \exp \left(i \int^t dt' \left(2\Delta_m - \frac{k_z}{a} \right) \right) \right|^2, \quad (7)$$

where \mathbf{k} is the comoving wavevector, $\mathcal{P}(k) \equiv |\delta_T(k)|^2$, $\delta_T(k)$ is the Fourier transform of the temperature fluctuation which we have assumed to be constant in time, a is the scale factor and z is defined to be the direction of neutrino propagation. In this derivation we have approximated the neutrino to be massless, therefore $ds^2 \simeq dt^2 - a(t)^2 dx^2 = 0$, and $|\Delta\mathbf{x}| = \int dt/a$. Note that the integral in Eq. (7) is oscillatory except when $2\Delta_m = k_z/a$, i.e. the fluctuation length is equal to the neutrino oscillation length. To solve the time integral we use the stationary phase approximation. Here we define new variables

$$Q(a, k_z) \equiv q(a) - k_z/h, \quad (8)$$

$$q(a) \equiv \frac{2a}{h} \Delta_m = \mu (a^2 + \lambda a^{-4}), \quad (9)$$

$$F(a) \equiv \frac{2Va \sin 2\theta_m}{h} = 2\mu\lambda \sin 2\theta, \quad (10)$$

$$\mu \equiv \frac{\Delta m^2}{2yT_0h}, \quad (11)$$

$$\lambda \equiv \frac{2\Delta C_v G_F^2 y T_0^6}{\alpha \Delta m^2 \cos 2\theta}, \quad (12)$$

$$(13)$$

where for temperatures of interest the universe is in the radiation dominated era, where $T = T_0/a$, $y = E/T$, the hubble parameter $H \equiv \dot{a}/a = h/a^2$ and T_0 , y and h are constant. We define $|I|^2$ such that $P = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \mathcal{P}(k) |I|^2$, applying the stationary phase approximation

$$|I|^2 = \left| \int da F(a) e^{i \int Q(a', k_z) da'} \right|^2 \quad (14)$$

$$\simeq \left| F(a_{s1}(k_z)) e^{iQ(a_{s1}(k_z), k_z)} \sqrt{\frac{2\pi}{iQ'(a_{s1}(k_z), k_z)}} + (a_{s1} \rightarrow a_{s2}) \right|^2 \quad (15)$$

$$\simeq \frac{2\pi F(a_{s1}(k_z))^2}{|Q'(a_{s1}(k_z), k_z)|} + (a_{s1} \rightarrow a_{s2}) + \frac{2\pi F(a_{s1}(k_z))F(a_{s2}(k_z))}{|Q'(a_{s1}(k_z), k_z)Q'(a_{s2}(k_z), k_z)|} \sin \int_{a_{s1}(k_z)}^{a_{s1}(k_z)} Q(a, k_z) da. \quad (16)$$

The integral in Eq. (14) is oscillatory except where $Q(k, a) = 0$, this defines a_{si} where $i=1, 2$, such that $Q(a_{si}, k_z(a_{si})) = 0$, note there are two real solutions to this equation. Once $|I|^2$ is integrated with respect to k_z the final term of Eq. (16) is averaged to zero and therefore it is no longer considered. Integrating $|I|^2$ with respect to k_z is done so by a change of variables, $dk_z = \frac{dk_z}{da_{si}} da_{si} = hQ'(a_{si}) ds_{si}$. Now the limits of integration are from $k_z = [-\infty, \infty]$, this corresponds to $a_{s1} = [0, a_0]$ and $a_{s1} = [a_0, \infty]$, where a_0 . Therefore the integral can be simplified to

$$P = \frac{h}{2\pi} \int_0^\infty dk_r \int_0^\infty da k_r F(a)^2 \mathcal{P}(\sqrt{k_r^2 + k_z(a)^2}), \quad (17)$$

where we have introduced the cylindrical polar co-ordinates (k_r, k_z, θ) . Note for a_{s1} there is a minus sign from $Q'(a_{s1}(k_z), k_z) = -|Q'(a_{s1}(k_z), k_z)|$ and also the limits of integration, where $k_{min} = a_0$ and $k_{max} = 0$. This results in a positive term in Eq (17). Now to calculate the level crossing probability we need the spectrum of the fluctuations.

3 Power Law Spectrum

The measured spectrum of fluctuations is approximately flat. This correlates to $\mathcal{P} = 2\pi^2 k^{-3} \Delta^2(k)$, where $\Delta^2(k) = \Delta^2(k_0)(k/k_0)^{n_s-1}$, and the spectral index $n_s - 1$ is small. From the combination of the WMAP3 [26] and SDSS [27] data, $n_s = 0.98 \pm 0.02$ at the 68% confidence level [28]. To first approximation we assume that the spectral index is constant. Our motivation for this is to analyze the effect of how the parameters effect the level crossing probability for the simplest case possible. For this case the level crossing probability is

$$P \simeq \left[\Delta^2(k_0) \left(\frac{h\mu \cos 2\theta \lambda^{1/3}}{k_0} \right)^{n_s-1} \right] \left[\frac{\mu \lambda^{1/2}}{\cos 2\theta} \right] \sin^2 2\theta \frac{4\pi}{2 - n_s} \int_0^{u_{max}} \frac{u^{8-4(n_s-1)}}{(1 + u^6)^{3-(n_s-1)}} du, \quad (18)$$

where we have defined $u \equiv \lambda^{-1/6} a$. The first square bracket is the size of the fluctuations at the neutrino oscillation scale with a scaling factor of $\lambda^{1/3}$. From Eq. (7) we can see that the fluctuations at the neutrino oscillation length scale is the important factor in increasing the level crossing probability. The second square bracket is the weighted number of oscillation lengths. This can be more easily seen if one expands $\mu \lambda^{1/2} = h^{-1}/(2yT_0/\Delta m^2 \lambda^{1/2})$ and realising that h^{-1} is the scaled Hubble length and $(2yT_0/\Delta m^2 \lambda^{1/2})$ is the scaled oscillation length. As one might naively expect the level crossing probability increases with the number of oscillation lengths. There is also a mixing factor $\sin^2 2\theta$ and a numerical factor which are as one would expect. Considering the oscillation of electron neutrinos into sterile neutrinos the oscillation probability is given by

$$P_{es} = 1/2 - (1/2 - P) \cos 2\theta_i \cos 2\theta_f, \quad (19)$$

where θ_i and θ_f are the mixing angles at production and detection of the neutrino. For small mixing $\cos 2\theta \simeq 1 - 2\theta^2$ and therefore

$$P_{es} \simeq (1 - 2P)(\theta_i^2 + \theta_f^2) + P. \quad (20)$$

For $P/\theta^2 \gtrsim 1$ the oscillation probability is dominated by the level crossing probability caused by the fluctuating plasma and is this dominant mechanism for the production of sterile neutrinos. In Figure 1 we plot the contours of constant P/θ^2 as a function of mass squared differences and spectral gradient. The level crossing probability increases with Δm^2 , this is because this corresponds to smaller oscillation length and therefore more oscillation periods. Also the level crossing probability increases with n_s , this is because the amplitude of the fluctuations on the neutrino oscillation length scale are larger.

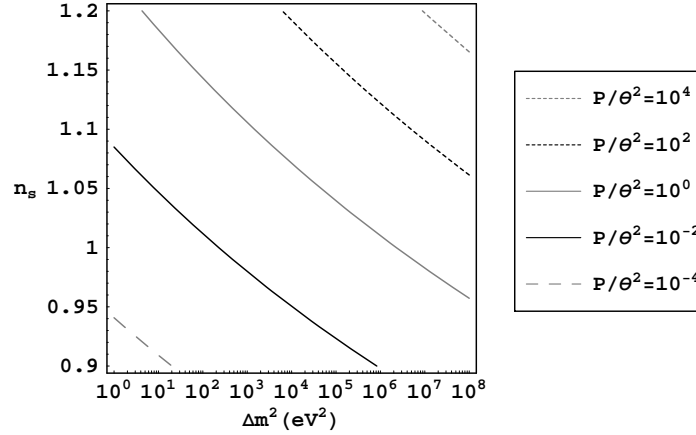


Figure 1: The contours of constant P/θ^2 as a function of spectral index $n_s - 1$ and difference in mass squared Δm^2 .

4 The primordial spectrum from Inflation

An initial stage of inflation seems a necessary ingredient of the Big Bang model[29]. An intriguing feature of inflationary models is that they predict a nearly scale invariant spectrum which is consistent with that observed by WMAP [26], the 2dFGRS [30] and SDSS [27]. A simple model capable of generating this spectrum of fluctuations for inflation is a single scalar field, ϕ , with the potential $V(\phi)$, in the slow role approximation $\varepsilon \equiv (M_{pl}V'/V)^2/2 \ll 1$ and $|\eta| \equiv M_{pl}^2|V''/V| \ll 1$, where $'$ represents $d/d\phi$. For this model the curvature fluctuations, $\Delta_R^2(k)$, which are related to the temperature fluctuations by $\mathcal{P}(k) = 2\pi^2k^{-3}\Delta_R^2(k)$ are given by

$$\Delta_R^2(k) = \frac{H^4}{(2\pi\dot{\phi})^2} \Big|_{t_*}, \quad (21)$$

where t_* is the time at which the k mode crossed the horizon, $k = aH|_{t_*}$. This together with the equation of motion for the inflaton field in the slow role approximation, $3H\dot{\phi} = -V'$, and the Hubble parameter, $H^2 = 8\pi V/3M_{pl}^2$ results in

$$\Delta_R^2(k) = \frac{128\pi}{9M_{pl}^2} \frac{V^3}{V'^2} \Big|_{t_*}. \quad (22)$$

Approximating H to be constant and using $k = aH|_{t_*}$

$$\ln(k/k_0) = - \int_{\phi_0}^{\phi} \frac{8\pi V d\phi}{M_{pl}^2 V'}. \quad (23)$$

For simplicity we consider the potential $V = a_n \phi^n$, for this case

$$\Delta_R^2(k) = \frac{128}{3n^2} \frac{V_0}{M_{pl}^4} \left(\frac{\phi_0}{M_{pl}} \right)^2 \left(1 - \frac{nM_{pl}^2}{4\pi\phi_0^2} \ln(k/k_0) \right)^{(2+n)/2}. \quad (24)$$

where $V_0 = a_n \phi_0^n$. We can see that the spectrum is flat for $n = 0, -2$, i.e. for $n_s = 1$, increases with k for $-2 < k < 0$, for $n_s > 1$ and decreases with k for $n > 0$ and $n < -2$, for $n_s < 1$. Now to obtain analytical results we can use observations of the primordial spectrum to determine three of the four free parameters. At the 68% confidence level $\ln(\Delta_R^2(k_0)10^{10}) = 3.17 \pm 0.06$ and $d \ln \Delta_R^2 / d \ln k|_{k_0} = n_s(k_0) - 1 = -0.02 \pm 0.02$, where $k_0 = 0.002 Mpc^{-1}$ [28], re-expressing Eq (24) in terms of these parameters

$$\Delta_R^2(k) = \Delta_R^2(k_0) \left(1 + \frac{n_s(k_0) - 1}{\alpha_n} \ln(k/k_0) \right)^{\alpha_n}, \quad (25)$$

where we have chosen $\alpha_n \equiv (2+n)/2$ to be undetermined³.

For this spectrum of primordial fluctuations the level crossing probability is

$$P = 2\pi \Delta_R^2 \mu \lambda^{1/2} \sin^2 2\theta \int_0^\infty dv \int_0^{u_m} du I_2 \quad (26)$$

$$I_2 = \frac{u^4 \left(v + (u^2 + u^{-4})^2 \right)^{-3/2}}{(1 + u^6)^2} \left(1 - \frac{1 - n_s(k_0)}{\alpha_n} \left(\ln \left(v + (u^2 + u^{-4})^2 \right) + \ln(h\mu\lambda^{1/3}/k_0) \right) \right)^{\alpha_n} \quad (27)$$

Figure 4 plots the contours of constant P/θ^2 for $n = -1$ for $n_s > 1$ and $n = 4$ for $n_s < 1$. The fluctuations increase with Δm^2 and n_s for as for the case of the power law spectrum. However the increase with n_s is not as pronounced as there is only a logarithmic increase with n_s . This mechanism becomes dominant in the production of sterile neutrinos for keV neutrinos with $n_s > 1$.

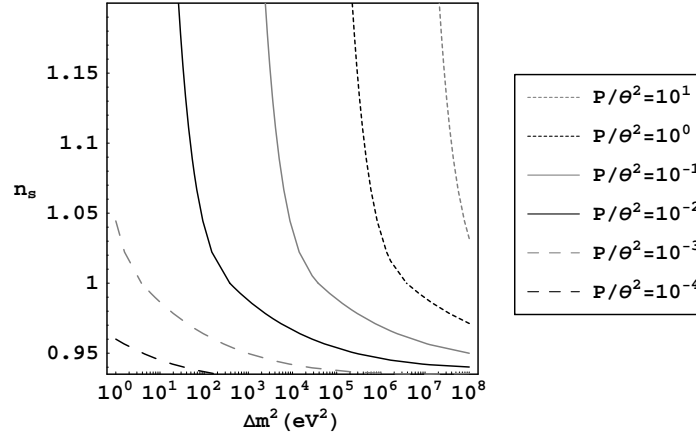


Figure 2: The contours of constant level crossing probability for a spectrum predicted by inflation with a potential $V = a_n \phi^n$ where $n = -1$ for $n_s > 1$ and $n = 4$ for $n_s < 1$.

³For fluctuations to be present at the neutrino oscillation length scale we require that the number of e-folds, $N_e = \int da/a \simeq 60 \lesssim \ln(k_0/k)$, for mass squared differences up to keV^2 this is true.

5 Neutrino Damping

So far we have neglected the damping of the fluctuations in the early universe by neutrino diffusion which is equivalent to Silk damping of photon decoupling. This occurs when the neutrinos diffuse from over dense regions dragging the matter with them. Neutrinos decouple at $T_{\nu_e}^{dec} \simeq 1.4 MeV$ for ν_e and $T_{\nu_\mu \nu_\tau}^{dec} \simeq 2.2 MeV$ for ν_μ and ν_τ [31], this occurs when universe expands faster than the neutrino interaction rate $\dot{H} > \Gamma_\nu$, where Γ_ν is the neutrino interaction rate. However, the neutrinos decouple from a particular mode of oscillation when $c_s k_{ph} > \Gamma_\nu$, where c_s is the sound speed of the oscillation. Following [32] the damping factor in the density fluctuations is

$$D(k, \eta) \equiv \exp(-\Gamma(k, \eta)) = \exp\left(-\frac{1}{2} \int_0^{\eta_{max}} (k_{phys}/\epsilon_{tot}) \eta_{visc} k d\eta\right) \quad (28)$$

where ρ_{tot} is the total energy density, $\eta_{visc} = (4/15)\Sigma \rho_{\nu_\alpha} \tau_{\nu_\alpha}$ is the shear viscosity co-efficient, ρ_{ν_α} is the energy density of ν_α , $\tau_{\nu_\alpha} \equiv 1/\Gamma_{\nu_\alpha}$ is the typical collision time and Γ_{ν_α} is the interaction rate of ν_α with the medium, $\eta_{max} = \min[\eta, \eta_{dec}(k)]$, $\eta_{dec}(k)$ is the comoving time at the decoupling of the oscillation mode k and the sum is over all active neutrino species. Substituting the neutrino interaction rate $\Gamma_\nu = \gamma_\nu G_F^2 T^5$, the energy density $\epsilon_\alpha = \pi^2 g_\alpha(T) T^4/30$, where g is the effective number of relativistic helicity degrees of freedom⁴ into Eq. (28)

$$\Gamma(k, T) = \sum_\nu \frac{4}{75} \frac{g_\nu}{g_{tot}} \left(\frac{H}{\Gamma_{\nu_\alpha}}\right)_{T_{max}} \left(\frac{k}{aH}\right)_{T_{max}}^2, \quad (29)$$

where $T_{max} = \max[T, T_\nu^{dec}(k)]$, $T_\nu^{dec}(k)$ decoupling temperature of ν with the oscillation mode k . The decoupling temperature of an oscillation mode can be related to the neutrino decoupling temperature by

$$T_\nu^{dec}(k) = T_\nu^{dec} \left(\frac{c_s k}{aH}\right)_{T_\nu^{dec}}^{1/4}, \quad (30)$$

where T_ν^{dec} is the ν decoupling temperature. For all k modes the neutrino has decoupled before the neutrino oscillation length is equal to the oscillation length (see appendix A) and therefore $T_{max} = T_{dec}(k)$ and the damping is maximal. This damping modifies the primordial spectrum to

$$\Delta^2(k) \rightarrow \Delta^2(k) \exp(-\Gamma(k)), \quad (31)$$

$$\Gamma(k) = \Lambda \left((u^2 + u^{-4})^2 + v\right)^{3/8}, \quad (32)$$

where

$$\Lambda \equiv \sum_\nu \frac{4}{75} \frac{g_\nu}{g_{tot}} \left(\frac{H}{\Gamma_{\nu_\alpha}}\right)_{T_{dec}} c_s^{-5/4} \left(\mu_{dec} \lambda_{dec}^{1/3} \cos 2\theta\right)^{3/4}, \quad (33)$$

where $\lambda_{dec} = \lambda a_{dec}^{-6}$, $\mu_{dec} = \mu a_{dec}^3$ and a_{dec} is the scale factor at the time of neutrino decoupling. This effectively gives a cutoff in the fluctuation spectrum when $\left((u^2 + u^{-4})^2 + v\right) = \Lambda^{-8/3} = 3.5 \times 10^{-10} \left(\frac{eV^2}{\Delta m^2}\right)^{4/3}$. As $\Lambda \propto \Delta(m^2)^{-4/3}$ the cutoff is larger for smaller mass differences. This is because smaller mass differences corresponds to a larger oscillation length, therefore fluctuations with smaller k . From Eq. (29) we see that damping is largest for (a) larger $k/\Gamma \sim \tau/L_{osc}$, where L_{osc} is the oscillation wavelength and (b) k/H . Physically (a) is due to the neutrinos 'dragging' the matter to distances larger than the oscillation wavelength and (b) is for smaller expansion rate and larger oscillation frequency there will be a larger number of oscillation lengths when the damping is large. The effect of this damping on the power spectrum with $n_s = 1$ is shown in Figure 3. Here we see that the damping severely reduces the level crossing probability for $\Delta m^2 \gtrsim 10^{-7}$. This damps any oscillations into sterile neutrinos in the $eV - keV$ mass range and the oscillations between active neutrinos.

⁴where fermionic degrees of freedom are suppressed by a factor of 7/8

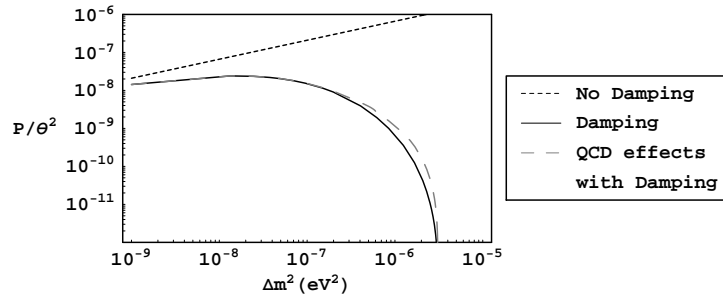


Figure 3: The effects of damping on the power law spectrum with and without fluctuation caused by the QCD phase transition.

6 QCD phase transition

At a temperature of about $T_* \sim 150 \text{ MeV}$ there is a QCD phase transition from a quark-gluon plasma to a hadron gas, this transition can increase the amplitude of the primordial fluctuations for sub horizon modes while leaving the superhorizon modes unaffected [33]. According to the second law of thermodynamics

$$\rho + p = T \frac{dp}{dT} \quad (34)$$

where ρ and p are the energy density and pressure respectively. During the phase transition the energy density is discontinuous in temperature at the temperature of the phase transition T_* , therefore the pressure must be continuous with a discontinuous gradient. During the phase transition the energy density decreases from $\rho_-(T_*)$ to $\rho_+(T_*)$, where we now use the notation $-$ and $+$ for the beginning and end of the transition respectively, the pressure p remains constant $p(T_*)$, and therefore the speed of sound $c_s = (\partial p / \partial \rho)_s^{1/2} = 0$. As the sound speed is zero there are no restoring forces from pressure gradients and therefore the radiation fluid goes into free fall. This can be seen quantitatively from the equations of motion for sub horizon density fluctuations

$$\delta' - k\hat{\psi} = 0 \quad (35)$$

$$\hat{\psi}' + c_s^2 \delta = 0 \quad (36)$$

where $\hat{\psi} = (\rho / (\rho + p)) v_{pec}$ and v_{pec} is the peculiar velocity. During the phase transition $\hat{\psi}' = 0$ and $\delta' = k\hat{\psi} = \text{const.}$ The solution to these equations are $\hat{\psi} = \hat{\psi}_-$ and $\delta = \delta_- + k(\eta - \eta_-)\hat{\psi}_-$, i.e. the energy density increases linearly with the time of the phase transition and the wavevector. This results in the ratio of the initial fluctuations, A_i , to the final oscillations, A_f being $A_f/A_i \simeq k/k_1$ where $k_1 = \sqrt{3}/(\eta_+ - \eta_-)$. This approximately modifies the spectrum of density fluctuations by

$$\Delta^2(\eta, k) \rightarrow \begin{cases} \Delta^2(\eta, k) & \text{for } k \leq k_1 \\ \Delta^2(\eta, k)(k/k_1)^2 & \text{for } k > k_1 \end{cases} \quad (37)$$

P/θ^2 is plotted in Figure 3 including the effects of the QCD phase transition. From this we see that the level crossing probability is increased but the fluctuations are still severely damped and consequently there is still no observable effect.

7 Conclusion

If sterile neutrinos mix with the standard model neutrinos they will be produced by oscillations in the early universe and the number produced will depend on the mass squared differences and mixing angles. Sterile

neutrinos would add to the relativistic energy density of the universe increasing the expansion rate of the universe resulting in an increase in the abundance of light elements. They would also contribute to the warm dark matter of the universe. Neutrino oscillations propagating in a fluctuating medium can be enhanced due to the increase in the level crossing probability between mass eigenstates. This probability is dominated by the amplitude of the fluctuations on the neutrino oscillation scale. The temperature fluctuations in the early universe have been probed by the WMAP, 2dFGRS and SDSS data and have shown that this spectrum of fluctuations is nearly scale invariant. In this paper we have calculated the level crossing probability for the case the spectrum of density fluctuations has a simple power law dependence and for the case this is modified by logarithmic running as predicted by a simple model of inflation. For these cases there is a region of parameter space for large Δm^2 and n_s where the production of sterile neutrinos would be dominated by this mechanism. However this conclusion ignores the damping of the fluctuation which occurs when the neutrinos decouple from the plasma, this is equivalent to Silk damping. We have shown that the neutrinos damp the fluctuations on the scale of the neutrino oscillation length scale and thus the level crossing probability is vastly reduced and is unobservable. A further effect is that the fluctuations on the neutrino oscillation scale are amplified at the QCD phase transition. However we find that this enhancement is insufficient to overcome the damping that occurs. In conclusion the production of sterile neutrinos does not get amplified from level crossing between the mass eigenstates due to primordial fluctuations because when the neutrinos decouple they damp the fluctuations at the neutrino oscillation scale.

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Appendix A

The decoupling temperature of the k oscillation mode is

$$T_\nu^{dec}(k) = T_\nu^{dec} \left(\frac{c_s k}{aH} \right)_{T_\nu^{dec}}^{1/4}. \quad (38)$$

The wavevector of the the neutrinos oscillation is

$$k = h\mu\lambda^{1/3} \cos 2\theta \sqrt{v + (u^2 + u^{-4})^2} \quad (39)$$

where $u = \lambda^{-1/6}a = \lambda^{-1/6}T/T_0$ where T is the temperature of the universe. The ratio of the decoupling temperature to the temperature of the universe is

$$\frac{T_\nu^{dec}(k)}{T} = \left(c_s \mu_{dec} \lambda_{dec} \cos 2\theta u^4 \sqrt{v + (u^2 + u^{-4})^2} \right)^{1/4}, \quad (40)$$

where $\lambda_{dec} = \lambda a_{dec}^{-6}$, $\mu_{dec} = \mu a_{dec}^3$ and a_{dec} is the scale factor at neutrino decoupling. The right hand side is minimized for $v = 0$ and $u = 0$, where at this minimum

$$\left. \frac{T_\nu^{dec}(k)}{T} \right|_{min} = \left((3/2^{2/3}) c_s \mu_{dec} \lambda_{dec} \cos 2\theta \right)^{1/4} \simeq 2.5. \quad (41)$$

As the minimum of this ratio is greater than unity the temperature of the universe when the wavelength of the fluctuation is equal to the neutrino oscillation length is always greater than the temperature at when the neutrinos decoupled from this mode.

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